Those problems marked with (+) will be collected in class for a grade. I will announce the due date one class period in advance. Those problems marked with $(*)$ denote problems that only those students taking this course as an honors course must complete. The remaining problems may be discussed in class and students should post solutions on the wiki.

1. Describe the set $\{m \in \mathbb{Z} \mid m^2 - m < 20\}$ by listing its elements.
2. (+) List the elements in $\{1, 2, c\} \times \{a, b, c\}$.
3. Let $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$. For each relation between A and B given as a subset of $A \times B$, decide whether it is a function mapping A into B. If it is a function, decide whether it is one-to-one and whether it is onto B. Justify your answers.

   (a) $\{(2, 4), (3, 6), (2, 2)\}$ (b) (+) $\{(1, 4), (2, 6), (3, 2)\}$ (c) $\{(1, 4), (2, 4), (3, 6)\}$

4. (+) Determine if the following relations define a function. If it is a function, determine whether it is one-to-one and onto. Justify your answers.

   (a) $f : \mathbb{R} \to \mathbb{Z}$, $f(x) = int(x)$, where $int(x)$ is the floor of x
   (b) $h : \mathbb{R} \to \mathbb{R}$, $h(t) = 2t + 6$

   For any set A, we denote by $\mathcal{P}(A)$ the collection of all subsets of A, called the **power set** of A.

5. List the elements of the power set of the given set and give the cardinality of the power set.

   (a) $\emptyset$ (b) $\{a\}$ (c) $\{a, b\}$ (d) $\{a, b, c\}$

6. (+) Let A be a finite set of order n. Based on the preceding exercise, make a conjecture about the value of $|\mathcal{P}(A)|$. Then try to prove your conjecture.
7. (*) For any set A, infinite or finite, let B^A be the set of all functions mapping A into the set $B = \{0, 1\}$. Show that the cardinality of B^A is the same as the cardinality of the set $\mathcal{P}(A)$. [*Hint*: Each element of B^A determines a subset of A in a natural way.]
8. Write the complex number $12 + 5i$ in polar form $|z|(p + qi)$ where $|p + qi| = 1$.
9. (+) Find all solutions in $\mathbb{C}$ of the given equations.

   (a) $z^4 = -1$ (b) $z^6 = 1$

10. Compute the following expressions using modular addition,

    (a) $8 +_{10} 6$ (b) (*) $2\sqrt{2} +_{\sqrt{32}} 3\sqrt{2}$

11. (+) Explain why the expression $5 +_6 8$ in $\mathbb{R}_6$ makes no sense.
12. Find all solutions x of the given equations.

    (a) (*) $x +_{2\pi} \frac{3\pi}{2} = \frac{3\pi}{4}$ in $\mathbb{R}_{2\pi}$ (b) (+) $x +_{12} = 2$ in $\mathbb{Z}_{12}$

13. Example 1.15 asserts that there is an isomorphism of U_8 with $\mathbb{Z}_8$ in which $\zeta = e^{i(\pi/4)} \leftrightarrow 5$ and $\zeta^2 \leftrightarrow 2$. Find the element of $\mathbb{Z}_8$ that corresponds to each of the remaining six elements ζ^m in U_8 for $m = 0, 3, 4, 5, 6$, and 7.
14. (+) There is an isomorphism of U_7 with $\mathbb{Z}_7$ in which $\zeta = e^{i(2\pi/7)} \leftrightarrow 4$. Find the element in $\mathbb{Z}_7$ to which ζ^m must correspond for $m = 0, 2, 3, 4, 5$, and 6.
15. (*) Why can there be no isomorphism of U_6 with $\mathbb{Z}_6$ in which $\zeta = e^{i(\pi/3)}$ corresponds to 4?