1. (+) Find the order of the following elements in the factor group.

   (a) $3 + \langle 4 \rangle$ in $\mathbb{Z}_{12}/\langle 4 \rangle$

   (b) $(3,3) + \langle (1.2) \rangle$ in $(\mathbb{Z}_4 \times \mathbb{Z}_8)/\langle (1,2) \rangle$

2. Classify the group $(\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}_8)/\langle (0,4,0) \rangle$ according to the fundamental theorem of finitely generated abelian groups.

3. (+) A student is asked to show that if H is a normal subgroup of an abelian group G, then G/H is abelian. The student's proof starts as follows:
   We must show that G/H is abelian. Let a and b be two elements in G/H.

   (a) Why does the instructor reading this proof expect to find nonsense from here on in the student's paper?

   (b) What should the student have written?

   (c) Complete the proof.

4. (+) Show that if H and N are subgroups of a group G, and N is normal in G, then $H \cap N$ is normal in H. Show by an example that $H \cap N$ need not be normal in G.

5. Show that the set of all $g \in G$ such that $i_g : G \to G$ is the identity inner automorphism is a normal subgroup of G.

6. Let $G = GL(n, \mathbb{R})$ and let K be a subgroup of $\mathbb{R}^*$. Prove $H = \{A \in G \mid \det A \in K\}$ is a normal subgroup of G.

7.

8. (+) Find the center and the commutator subgroup of $\mathbb{Z}_3 \times S_3$

9. To what group mentioned in the text is $\mathbb{R}/\mathbb{Z}$ isomorphic?

10. Let $H, K \trianglelefteq G$. Give an example showing we may have $H \simeq K$ while G/H is not isomorphic to G/K.

11. (+) Describe the center of every *simple*

    (a) abelian group

    (b) nonabelian group

12. (*) Prove that A_n is simple for $n \geq 5$ by following the setps outlined in problem 39 of section 15 in the book.