Those problems marked with (+) will be collected in class for a grade. I will announce the due date one class period in advance. Those problems marked with (∗) denote problems that only those students taking this course as an honors course must complete. The remaining problems may be discussed in class and students should post solutions on the wiki.

1. (+) Let S be the set of real numbers, $S = \mathbb{R}$. If $a, b \in S$, define $a \sim b$ if $a - b$ is an integer. Show that $\sim$ is an equivalence relation on S. Describe the equivalence classes of S.

2. (+) Consider the binary operation $*$ defined on $S = \{a, b, c, d, e\}$ by the table below.

$*$	a	b	c	d	e
a	a	b	c	b	d
b	b	c	a	e	c
c	c	a	b	b	a
d	b	e	b	e	d
e	d	b	a	d	c

   (a) Compute $b * d, c * c$, and $[(a * c) * e] * a$.

   (b) Compute $(a * b) * c$ and $a * (b * c)$. Can you say on the basis of these computations whether $*$ is associative?

   (c) Compute $(b * d) * c$ and $b * (d * c)$. Can you say on the basis of these computations whether $*$ is associative?

   (d) Is $*$ commutative? Why?

3. Complete the table below so as to define a *commutative operation* $*$ on $S = \{a, b, c, d\}$.

$*$	a	b	c	d
a	a	b	c	
b	b	d		c
c	c	a	d	b
d	d			a

4. (+) Determine whether the binary operation $*$ defined on $\mathbb{Z}$ by letting $a * b = a - b$ is commutative. Is $*$ associative?

5. Determine whether the binary operation $*$ defined on $\mathbb{Q}$ by letting $a * b = ab/2$ is commutative. Is $*$ associative?

6. (+) Let S be a set having exactly one element. How many different binary operations can be defined on S? Answer the question if S has exactly 2 elements; exactly 3 elements; exactly n elements.

7. (*) How many different commutative binary operations can be defined on a set of 2 elements? on a set of 3 elements? on a set of n elements?

8. Determine if the definition of $*$ does give a binary operation on the set. If it does not, state whether condition 1, condition 2, or both conditions given in class are violated.

   (a) (+) On $\mathbb{Z}^+$, define $*$ by $a * b = a - b$

   (b) On $\mathbb{Z}^+$, define $*$ by $a * b = c$, where c is the largest integer less than the product of a and b.

9. (+) Let H be the subset of $M_2(\mathbb{R})$ consisting of all matrices of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ for $a, b \in \mathbb{R}$. Is H closed under

(a) matrix addition? (b) matrix multiplication?

10. (+) Give a set different from any of those described in the examples of the text and not a set of numbers. Define two different binary operations $*$ and $*'$ on this set. Be sure that your set is well-defined.

11. (+) Prove or give a counterexample: Every binary operation on a set consisting of a single element is both commutative and associative.

12. (+) Let F be the set of all real-valued functions having as domain $\mathbb{R}$. Prove or give a counterexample for the following statements.

    (a) Function addition $+$ on F is associative.

    (b) Function subtraction $-$ on F is commutative.

13. (*) Suppose that $*$ is an associative and commutative binary operation on a set S. Show that $H = \{a \in S \mid a * a = a\}$ is closed under $*$. (The elements of H are **idempotents** of the binary operation $*$.)

14. Determine whether the given relation is an equivalence relation. If it is, describe the partition arising from the equivalence relation.

    (a) $n\mathcal{R}m$ in $\mathbb{Z}$ if $nm > 0$ (b) $a\mathcal{R}b$ in $\mathbb{Z}$ if $a + b$ is even