Those problems marked with (+) will be collected in class for a grade. I will announce the due date one class period in advance. Those problems marked with ($*$) denote problems that only those students taking this course as an honors course must complete. The remaining problems may be discussed in class and students should post solutions on the wiki.

1. (+) Determine if the following sets of invertible $n \times n$ matrices with real entries defines a subgroup of $GL(n, \mathbb{R})$.
   (a) $\{A \in GL(n, \mathbb{R}) \mid \det A = \pm 1\}$
   (b) For $n = 2$, the set of upper triangular invertible matrices. (If $A = [a_{ij}]$, $a_{ij} = 0$ for $i > j$.)
2. Write at least five elements of each of the following cyclic groups.
   (a) $14\mathbb{Z}$ under addition
   (b) $\langle \frac{2}{3} \rangle = \{ \frac{2}{3}^n \mid n \in \mathbb{Z}\}$
3. (+) Describe all of the elements in the cyclic subgroup of $GL(2, \mathbb{R})$ generated by the matrix $\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$.
4. Which of the following groups are cyclic? For each cyclic group, list all the generators of the group.
   (a) $G_1 = \langle \mathbb{Z}, + \rangle$
   (b) $G_2 = \{a + b\sqrt{5} \mid a, b \in \mathbb{Z}\}$ under addition
   (c) $G_3 = \langle \mathbb{Q}^+, \cdot \rangle$
   (d) $G_4 = \langle \mathbb{Q}, + \rangle$
   (e) $G_5 = \langle 7\mathbb{Z}, + \rangle$
   (f) $G_6 = \{7^n \mid n \in \mathbb{Z}\}$ under multiplication
5. Find the order of the cyclic subgroup of the given group listed below.
   (a) (+) The subgroup of $\mathbb{Z}_9$ generated by 6
   (b) (+) The subgroup of V generated by c
   (c) The subgroup of U_5 generated by $\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$
6. (a) (+) Create a table for the group $\mathbb{Z}_8$ under addition.
   (b) (+) Compute the subgroups $\langle 0 \rangle, \langle 1 \rangle, \langle 2 \rangle, \langle 3 \rangle, \langle 4 \rangle, \langle 5 \rangle, \langle 6 \rangle, \langle 7 \rangle$.
   (c) (+) Which elements are generators for the group $\mathbb{Z}_8$?
   (d) (+) Give the subgroup diagram for the part (b) subgroups of $\mathbb{Z}_8$.
7. Mark each of the following true or false.
   (a) The associative law holds in every group.
   (b) $\mathbb{Z}_5$ is a cyclic group.
   (c) Every group is a subgroup of itself.
   (d) In every cyclic group, every element is a generator.
   (e) Every set of numbers that is a group under addition is also a group under multiplication.
   (f) Every subset of every group is a subgroup under the induced operation.
   (g) A subgroup may be defined as a subset of a group.
8. (+) Let $\phi : G \to G'$ be an isomorphism of a group $\langle G, * \rangle$ with the group $\langle G', *' \rangle$. Prove: If H is a subgroup of G, then $\phi[H] = \{\phi(h) \mid h \in H\}$ is a subgroup of G'. That is, an isomorphism carries subgroups into subgroups.
9. (*) (a) Show that a nonempty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$ for all $a, b \in H$.
   (b) Let H be a subgroup of a group G. For $a, b \in G$, define $a \sim b$ if and only if $ab^{-1} \in H$. Show that $\sim$ defines an equivalence relation on G.

10. If G is a group, the **center** of G is defined to be $Z(G) = \{x \in G \mid xa = ax \text{ for all } a \in G\}$. Show that $Z(G)$ is a subgroup of G.

11. (*) Suppose that H is a nonempty subset of a group G that is closed under the group operation and has the property that if a is not in H then a^{-1} is not in H. Is H a subgroup?

12. (+) If H and K are subgroups of G, show that $H \cap K$ is a subgroup of G.