1. (+) Let G be a group and $a \in G$.
   (a) If $a^{12} = e$, what can we say about $|a|$?
   (b) If $a^m = e$, what can we say about $|a|$?
   (c) Suppose that $|G| = 24$ and that G is cyclic. If $a^8 \neq e$ and $a^{12} \neq e$, show that $\langle a \rangle = G$.
2. Let $G = \langle a \rangle$ be a cyclic group and G' a group isomorphic to G. IF $\phi : G \to G'$ is an isomorphism, show that for every $x \in G$, $\phi(x)$ is completely determined by $\phi(a)$. (i.e., if $\phi, \psi : G \to G'$ are both isomorphisms such that $\phi(a) = \psi(a)$, then $\phi(x) = \psi(x)$ for all $x \in G$.)
3. Show that $\mathbb{Z}_p$ has no proper subgroups if p is prime.
4. (*) Let p and q be distinct primes.
   (a) Find the number of generators of the cyclic group $\mathbb{Z}_{pq}$.
   (b) Find the number of generators of the cyclic group $\mathbb{Z}_{p^r}$, where $r \in \mathbb{Z}^+$.
5. An **automorphism of a group** is an isomorphism of the group with itself. Find the number of automorphisms of the following groups. (Hint: Make use of number 2. What must the image of a generator be under an automorphism?)
   (a) $\mathbb{Z}_6$ (b) (+) $\mathbb{Z}_{20}$
6. Prove or disprove: If every proper subgroup of a group G is cyclic, then G is also cyclic.
7. (+) True or False? An element a of a group G has order $n \in \mathbb{Z}^+$ if and only if $a^n = e$.
8. (+) Let G be an abelian group and let $H = \{g \in G \mid |g| divides 12\}$. Prove that H is a subgroup of G. Would your proof be valid if the number 12 were replaced by another number? State the general result.