1. (+) Let
$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{pmatrix} \qquad \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}$$
Compute the following
   (a) α^{-1}
   (b) $\alpha\beta$
   (c) $\beta\alpha$
   (d) α^2

2. Write the following permutations as (a) a product of disjoint cycles; (b) a product of transpositions
   (a) (+) (1235)(413)
   (b) (13256)(23)(46512)
   (c) (12)(13)(23)(142)

3. (a) (+) Prove that the order of a cycle of length k is k.
(b) (+) Prove that the order of the product of two disjoint cycles in S_n ($n \geq 2$) is the least common multiple of the lengths of the cycles. Deduce that the order of a product of m disjoint cycles is the least common multiple of their lengths.
(c) (*) Write out the possible disjoint cycle structures of S_7. (For ease of notation, let $(\underline{n})$ denote a cycle of length n, so for example, $(\underline{4})(\underline{2})(\underline{1})$ is one possible structure, which will have order 4.) Determine the possible orders of elements of S_7.
(d) (+) Find the orders of the following elements of S_7: (a) (135) (b) (24)(163) (d) (124)(3576) (e) (1234)(175)

4. Show that A_8 contains an element of order 15.

5. How many elements of order 5 are there in S_6?

6. In S_4, find a cyclic subgroup of order 4 and a noncyclic subgroup of order 4.

7. (+) Find the maximum possible order for an element of S_{10}.

8. (*) Find eight elements in S_6 that commute with (12)(34)(56). Do they form a subgroup of S_6?

9. Consider a regular plane n-gon for $n \geq 3$. (We did this for $n = 3, 4$ in class.) Each way such an n-gon can be placed, with one covering the other, corresponds to a certain permutation of the vertices. The set of these permutations is the n^{th} dihedral group D_n under permutation multiplication. Find the order of the group D_n. Argue *geometrically* that this group has a subgroup having just half as many elements as the whole group has.

10. (*) Show that for every subgroup H of S_n for $n \geq 2$, either all the permutations in H are even or exactly half of them are even.

11. (+) Let G be a group and let $a \in G$ be a fixed element of G. Show that the map $\lambda_a : G \to G$, given by $\lambda_a(g) = ag$ for all $g \in G$, is a permutation of the set G.

12. (+) Let G be a group of permutations on a set A. Let $a \in A$ and define $\text{stab}(a) = \{\alpha \in G \mid \alpha(a) = a\}$. Then $\text{stab}(a)$ is called the **stabilizer of** a **in** G. Prove that $\text{stab}(a)$ is a subgroup of G.

13. Prove if $\sigma, \tau \in S_n$, then $\sigma \circ \tau$ is an even permutation if both σ, τ are even or both are odd.