"I don't know, Marge. Trying is the first step towards failure." - Homer Simpson

1. (+) (a) Let $H = \{(1), (12)(34), (13)(24), (14)(23)\}$. Find the left cosets of H in A_4.
   (+) (b) Let H be as above. How many left cosets of H are there in S_4? (Calculate this *without* finding all the cosets.)
2. (+) (a) Find all the left cosets of the subgroup $\{\rho_0, \mu_2\}$ of D_4. (Table 8.12 gives the multiplication table for D_4.)
   (+) (b) Now find the right cosets. Are they the same as the left cosets?
3. Find all the left cosets of the subgroup $\langle 3 \rangle$ of $\mathbb{Z}_{12}$.
4. (+) Find the index of $\langle 4 \rangle$ in $\mathbb{Z}_{24}$.
5. Find the index of $\sigma = (456)(13)$ in S_6.
6. Let $|a| = 30$. How many left cosets of $\langle a^4 \rangle$ in $\langle a \rangle$ are there?
7. (+) Let G be a group with $|G| = pq$, where p and q are prime. Prove that every proper subgroup of G is cyclic.
8. Let G be a group of order 60. What are the possible orders of subgroups of G?
9. (+) Let $|G| = 33$. What are the possible orders of an element of G? Show that G must contain an element of order 3.
10. (*) Suppose H and K are subgroups of a group G. If $|H| = 12$ and $|K| = 35$, find $|H \cap K|$. Generalize.
11. (*) Let $|G| = 8$. Show that G must have an element of order 2.
12. (a) Suppose $|G| = 2n$, $n \in \mathbb{Z}$. Show that G contains an element of order 2. (i.e., $\exists a \in G$ such that $a * a = e$.)
    (b) Use Lagrange's Theorem to show that if G is abelian and $|G| = 2n$ where n is odd, then G contains exactly one element of order 2.
13. (+) Let H be a subgroup of a group G with the property that $g^{-1}hg \in H$ for all $g \in G$ and all $h \in H$. Show that every left coset gH is equal to the right coset Hg.