

1. (+) List the elements of $\mathbb{Z}_2 \times \mathbb{Z}_4$, along with each of their orders. Is the group cyclic?
2. (*) What is the largest order among the orders of all cyclic subgroups of $\mathbb{Z}_{15} \times \mathbb{Z}_{25}$?
3. Find all abelian groups, up to isomorphism of the following orders.
 - (a) 81
 - (b) (+) 2200
4. Mark the following as true or false.
 - (a) \mathbb{Z}_8 is generated by $\{4, 6\}$.
 - (b) Every abelian group of order divisible by 5 contains a cyclic subgroup of order 5.
 - (c) $\mathbb{Z}_2 \times \mathbb{Z}_4$ is isomorphic to \mathbb{Z}_8 .
 - (d) $\mathbb{Z}_2 \times \mathbb{Z}_4$ is isomorphic to \mathbb{Z}_8 .
 - (e) The order of $\mathbb{Z}_{12} \times \mathbb{Z}_{15}$ is 60.
5. (+) Show that there are two Abelian groups of order 108 that have exactly one subgroup of order 3.
6. Show that there are two Abelian groups of order 108 that have exactly four subgroups of order 3.
7. (+) The group $\{1, 9, 16, 22, 29, 53, 74, 79, 81\}$ is a group under multiplication modulo 91. Determine the isomorphism class of this group.
8. (*) Up to isomorphism, how many Abelian groups of order 16 have the property that $x + x + x + x = 0$ for all x in the group?
9. (+) Without using Lagrange's Theorem, show that an Abelian group of odd order cannot have an element of even order.