

Read each question carefully. Be sure to answer completely and show all of your work.

1. Find all the left cosets of $\langle(12)\rangle$ in \mathbb{S}_3 .

2. Let G be a group and $H \leq G$. Suppose $a \in G$. Prove that $aH = H$ if and only if $a \in H$.

3. (a) Use the Fundamental Theorem of Finitely Generated Abelian Groups to list all abelian groups of order 108, up to isomorphism.

(b) Show that there are two abelian groups of order 108 that have exactly four subgroups of order 3.