

Choose four of the following six problems to answer. Each problem is worth 10 points. Be sure to answer each question completely and explain your reasoning. Write your solutions on separate paper. You may refer to your book, notes, and the wiki as you work on this, but you may not discuss the problems with anyone else or use any online sources. By signing your name above, you acknowledge that you have adhered to the rules outlined here.

1. The set $\mathbb{Z}_{45}^* = \{1, 4, 11, 14, 16, 19, 26, 29, 31, 34, 41, 44\}$ is a group under multiplication modulo 45. Express \mathbb{Z}_{45}^* as a direct product of groups. (That is, find the isomorphism class of \mathbb{Z}_{45}^* .)
2. (a) If H and K are subgroups of a group G and $g \in G$, show $g(H \cap K) = gH \cap gK$.
(b) If H and K are subgroups of a group G and $a, b \in G$ such that $aH \subseteq bK$, prove that $H \subseteq K$.
3. (a) How many homomorphisms are there from \mathbb{Z}_{20} onto \mathbb{Z}_8 ? How many are there to \mathbb{Z}_8 ?
(b) Find a homomorphism $\phi : \mathbb{Z}_{40}^* \rightarrow \mathbb{Z}_{40}^*$ with kernel $\{1, 9, 17, 33\}$ and $\phi(11) = 11$.
4. Suppose H and K are distinct subgroups of G of index 2. Prove that $H \cap K$ is a normal subgroup of G of index 4 and that $G/(H \cap K)$ is not cyclic.
5. Let R be a ring. The center of R is the set $\{x \in R \mid ax = xa \text{ for all } a \in R\}$. Prove that the center of a ring is a subring.
6. (a) Find all the zero divisors in the ring \mathbb{Z}_{30} .
(b) Let R be a ring with unity 1. If R has no zero divisors, prove that $ab = 1$ implies $ba = 1$.

(10 points) For the homework and section(s) that you have been assigned, make sure that all definitions and theorems are correct and complete on the wiki. Similarly, make sure that the homework is complete and correct. If you feel your assignment is unusually long, please email me. If you believe that there is nothing to fix on your assignment, email me to receive a new assignment. You may receive help from your classmates or me with the edits to the wiki. You should list your edits on the wiki under contributions. Your assignment is listed below.

- Bryson - HW1, Sections 0-1
- Kerry - HW2, Section 2
- Bingxin - HW3, Section 3
- Dustin - HW4, Section 4
- Alexander - HW5, Section 5
- Hunter - HW6, Section 6
- Joel - HW7, Sections 8-9
- Tiffani - HW8, Section 10
- Katelyn - HW9, Section 11
- Siyu - HW10, Section 13
- Sean - HW 11, Sections 14-15
- Cody - HW12, Sections 18-19