1. Draw the subgroup diagram for $\mathbb{Z}_{16}$.
2. List the elements in the cyclic subgroup generated by (168)(235) in S_8.
3. For any a, b in a group and any $n \in \mathbb{Z}$, show that $(a^{-1}ba)^n = a^{-1}b^n a$.
4. Let G be a group with the property that $\forall x, y, z \in G$, $xy = zx$ implies $y = z$. Prove that G is abelian.
5. Let S be the set of polynomials with real coefficients. Define $f \sim g$ for $f, g \in S$ if $f' = g'$ (that is, their derivatives are equal). Show that $\sim$ defines an equivalence relation on S. Describe the equivalence class of f.
6. If a, b are group elements with $|a| = 6$ and $|b| = 7$, express $(a^4c^{-2}b^4)^{-1}$ without using negative exponents.
7. If H and K are subgroups of a group G, show that $H \cap K \leq G$.
8. In $\mathbb{Z}_{24}$, find a generator for $\langle 21\rangle \cap \langle 10\rangle$. Suppose $|a| = 24$ in a group G. Find a generator for $\langle a^{21}\rangle \cap \langle a^{10}\rangle$.
9. Suppose G is a cyclic group with exactly 3 subgroups: G itself, $\{e\}$, and a subgroup of order 7. What can you say about $|G|$?
10. Write $\sigma = (13256)(23)(46512)$ as (a) a product of disjoint cycles; (b) a product of transpositions. (c) Is $\sigma \in A_6$?
11. Let $\beta = (123)(145)$. Write β^{99} in disjoint cycle form.
12. Let $H = \{\beta \in S_5 \mid \beta(1) = 1 \text{ and } \beta(3) = 3\}$. Prove $H \leq S_5$. How many elements are in H?
13. Let $\phi : G \to G$ be a group automorphism. Prove $H = \{x \in G \mid \phi(x) = x\}$ is a subgroup of G.
14. Let $|a| = 30$. How many left cosets of $\langle a^4\rangle$ are there in $\langle a\rangle$?
15. List the elements of the factor group $\mathbb{Z}_{24}/\langle 8\rangle$. What is the order of the element $14 + \langle 8\rangle$ in this group?
16. Let $G = \mathbb{Z}_4 \times \mathbb{Z}_4$, $H = \{(0,0), (2,0), (0,2), (2,2)\}$ and $K = \langle(1,2)\rangle$. Classify G/H and G/K according to the Fundamental Theorem of finitely generated Groups.
17. Prove $(A \times B)/(A \times \{e\}) \simeq B$.
18. If ϕ is a homomorphism from $\mathbb{Z}_{30}$ onto a group of order 5, determine the kernel of ϕ.
19. Give an example of a subset of a ring that is a subgroup under addition, but is not a subring.
20. The ring $\{0, 2, 4, 6, 8\}$ under multiplication modulo 10 has unity. Find it. Show this ring is a field.
21. List all zero divisors in $\mathbb{Z}_{20}$.
22. Find all zeros of $x^3 + 2x + 2$ in $\mathbb{Z}_7$.
23. Let R be a ring, and let a be a fixed element of R. Let $I_a = \{x \in R \mid ax = 0\}$. Show that I_a is a subring of R.
24. Is $2x^3 + x^2 + 2x + 2$ an irreducible polynomial in $\mathbb{Z}_5[x]$? Why or why not? Express it as a product of irreducible polynomials in $\mathbb{Z}_5[x]$.
25. Determine if the polynomial $4x^{10} - 9x^4 + 24x - 18$ is irreducible over $\mathbb{Q}$.